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Stability Analysis and Optimal Harvesting Policy of Prey-Predator Model with Stage Structure for Predator

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Abstract

In this paper, we study a prey-predator model with stage structure for predator and constant harvesting efforts of the prey and predator population. The existence and stability of interior equilibrium point are analyzed. The maximum profit problem is solved and optimal harvesting policy of present value of revenue is solved using Pontryagin's maximum principle. From the analysis, there exists a critical value of the efforts that maximizes the profit and the present value of the revenue and the equilibrium point is also remain stable. Finally, numerical examples are given.

Keywords: Prey-predator, stage structure, harvesting, profit, present value

1 Introduction

The prey-predator population model is one of the most popular models in mathematical ecology. The prey-predator model has been extended by many authors

with attention to aspects of harvesting, time delay, stage structure, diffusion, and many more variations. Some researchers have considered two stage, immature and mature, in the prey population or in the predator population, for examples in [1, 2, 3]. Prey-predator model with stage structure and harvesting has been investigated intensively in some perspective. In [4], the prey population is classified in two sub population, immature and mature prey, with harvesting policy in maximum sustainable yield, while in [5], the prey-predator model with stage structure and impulsive harvesting is considered. Some authors have considered the prey-predator population with stage structure and selective harvesting for prey, (for examples in [4, 6]), selective harvesting for predator (for examples in [7, 8, 9, 10]), and selective harvesting for adult prey and predator (for example in [11]).

Based on the works of the authors above, we present a prey-predator model with stage structure for predator and constant efforts of harvesting for all populations (prey, immature predator, and mature predator). The objective of this paper is to study the effects of harvesting on the dynamic of prey-predator model. Besides that, for the model with constant efforts of harvesting we relate the stable equilibrium point to the maximum present value of a continuous time-stream of revenue by using Pontryagin's maximum principle.

2 The Prey-Predator Model with Stage Structure

The prey-predator model that we consider involves three populations (prey, immature predator, and mature predator). The model includes two predators with stage structure. In [6], prey-predator model with stage structure in predator has been analyzed but different in some assumptions. The considered model is

$$\begin{aligned}\frac{dN_1}{dt} &= r_1 N_1 \left(1 - \frac{N_1}{K}\right) - \alpha_1 N_1 N_3 \\ \frac{dN_2}{dt} &= -r_2 N_2 + \beta_2 N_3 - \gamma_2 N_2 \\ \frac{dN_3}{dt} &= -r_3 N_3 + \gamma_2 N_2 + m \alpha_1 N_1 N_3.\end{aligned}\quad (1)$$

$N_1 = N_1(t)$ denotes population size of prey at time t , $N_2 = N_2(t)$ denotes population size of immature predator at time t . $N_3 = N_3(t)$ denotes population size of mature predator at time t . Parameter r_1 is the intrinsic growth of prey population, K is the carrying capacity of environment for prey population, α_1 measures the rate of consumption of prey by the mature predator, and m ($0 < m < 1$) is the weighted of predation to the mature predator. Parameter r_2 and

r_3 are the mortality rate for immature and mature predator respectively. Parameter β_2 is the birth rate of predator population and γ_2 is the conversion rate from immature predator to mature predator.

We assume that the populations in the model are beneficial for human, therefore the populations are harvested. In this model, we assume the populations are harvested with constant efforts. The model (1) is then extended and becomes

$$\begin{aligned}\frac{dN_1}{dt} &= r_1 N_1 \left(1 - \frac{N_1}{K}\right) - \alpha_1 N_1 N_3 - q_1 E_1 N_1 \\ \frac{dN_2}{dt} &= -r_2 N_2 + \beta_2 N_3 - \gamma_2 N_2 - q_2 E_2 N_2 \\ \frac{dN_3}{dt} &= -r_3 N_3 + \gamma_2 N_2 + m\alpha_1 N_1 N_3 - q_3 E_3 N_3.\end{aligned}\quad (2)$$

Parameters q_1 , q_2 , and q_3 in model (2) denote the catchability coefficient for the prey and predator respectively. Parameters E_1 , E_2 , and E_3 denote the efforts of harvesting and we also assume $0 \leq E_i \leq E_{i\max}$ for $i = 1, 2, 3$. Let $r_4 = r_1/K$, $q_4 = r_1 - q_1 E_1$, $q_5 = r_2 + \gamma_2 + q_2 E_2$, $q_6 = r_3 + q_3 E_3$, and $\alpha_3 = m\alpha_1$, then model (2) becomes

$$\begin{aligned}\frac{dN_1}{dt} &= N_1 (q_4 - r_4 N_1 - \alpha_1 N_1) \\ \frac{dN_2}{dt} &= -q_5 N_2 + \beta_2 N_3 \\ \frac{dN_3}{dt} &= -q_3 N_3 + \gamma_2 N_2 + \alpha_3 N_1 N_3.\end{aligned}\quad (3)$$

There are three non negative equilibrium points in model (3), namely

$$T_1 = (0, 0, 0), \quad T_2 = \left(\frac{q_4}{r_4}, 0, 0\right), \quad \text{and} \quad T_3 = (N_1^*, N_2^*, N_3^*), \quad \text{where} \quad N_1^* = \frac{p_5}{\alpha_3 q_5},$$

$$N_2^* = \frac{\beta_2 p_6}{\alpha_3 q_5^2 \alpha_1}, \quad N_3^* = \frac{\beta_2 p_6}{\alpha_3 q_5^2 \alpha_1}, \quad p_5 = q_5 q_6 - \beta_2 \lambda_2, \quad \text{and} \quad p_6 = q_4 \alpha_3 q_5 - p_5 r_4.$$

The equilibrium point T_3 is the only interior equilibrium point when $p_5 = q_5 q_6 - \beta_2 \lambda_2 > 0$ and $p_6 = q_4 \alpha_3 q_5 - p_5 r_4 > 0$. The Jacobian matrix of model (3) at the equilibrium point $T_3 = (N_1^*, N_2^*, N_3^*)$ is given by

$$M_E = \begin{pmatrix} q_4 - 2r_2N_1^* - \alpha_1N_3^* & 0 & -\alpha_1N_1^* \\ 0 & -q_5 & \beta \\ \alpha_3N_3^* & \gamma_2 & -q_6 + \alpha_3N_1^* \end{pmatrix}.$$

The characteristic equation of Jacobian matrix at T_3 is given by

$f(\rho) = \det(\rho I - M_E)$, i.e. $f(\rho) = \rho^3 + a_2\rho^2 + a_1\rho + a_0$, where

$$a_2 = -\alpha_3N_1^* + 2r_4N_1^* + \alpha_1N_3^* + q_6 + q_5 - q_4,$$

$$a_1 = q_6q_5 - q_5\alpha_3N_1^* + \alpha_1N_3^*q_6 + \alpha_1N_3^*q_5 - q_4q_5 - 2r_4N_1^{*2}\alpha_3 + q_4\alpha_3N_1^* - q_4q_6 - q_4q_6 + 2r_4N_1^*q_6 + 2r_4N_1^*q_5 - \gamma_2\beta_2,$$

$$a_0 = -2r_4N_1^*\gamma_2\beta_2 - q_4q_5q_6 + q_4q_5\alpha_3N_1^* - 2r_4N_1^{*2}q_5\alpha_3 - \alpha_1N_3^*\gamma_2\beta_2 + q_4\gamma_2\beta_2 + \alpha_1N_3^*q_5q_6 + 2r_4N_1^*q_5q_6.$$

Following the Routh-Hurwitz criteria [12], the equilibrium point T_3 is asymptotically stable when $a_0 > 0$, $a_2 > 0$, and $a_2a_1 - a_0 > 0$.

3 Bionomic Equilibrium

The concept of bionomic equilibrium that integrates the biological equilibrium and the economic equilibrium has been considered in [13]. The biological equilibrium is found by solving $\frac{dN_1}{dt} = 0$, $\frac{dN_2}{dt} = 0$, and $\frac{dN_3}{dt} = 0$ simultaneously. The economic equilibrium is reached when the total revenue from selling harvested biomass equals to the total cost of harvesting efforts.

We assume that the total cost are directly proportional to the fishing effort, $TC = C(E) = cE$. Total revenue gained by selling (fish) catch is directly proportional to the yield, $TR = pY(E)$, where $Y(E) = EqN$ represents the sustained yield function which has been considered in [14]. Further, the profit function is $\pi = TR - TC$. Let c_i denotes harvesting cost per unit effort of population N_i and p_i denotes the price per unit biomass of population N_i . The profit function is given by

$$\pi = (p_1q_1N_1)E_1 + (p_2q_2N_2)E_2 + (p_3q_3N_3)E_3 - (c_1E_1 + c_2E_2 + c_3E_3).$$

The bionomic equilibrium $(N_1^*, N_2^*, N_3^*, E_1^*, E_2^*, E_3^*)$ is found by solving the following simultaneous equation

$$\begin{aligned} N_1(q_4 - r_4 N_1 - \alpha_1 N_1) &= 0 \\ -q_5 N_2 + \beta_2 N_3 &= 0 \\ -q_3 N_3 + \gamma_2 N_2 + \alpha_3 N_1 N_3 &= 0 \\ (p_1 q_1 N_1) E_1 + (p_2 q_2 N_2) E_2 + (p_3 q_3 N_3) E_3 - (c_1 E_1 + c_2 E_2 + c_3 E_3) &= 0. \end{aligned}$$

Now, we relate the positive equilibrium point $T_3 = (N_1^*, N_2^*, N_3^*)$ to the maximum profit problem. The equilibrium point $T_3 = (N_1^*, N_2^*, N_3^*)$ is an interior equilibrium point if

$$(r_3 + q_3 E_3)(r_2 + \gamma_2 + q_2 E_2) - \gamma_2 \beta_2 > 0 \tag{4}$$

and

$$\alpha_3 (r_2 + \gamma_2 + q_2 E_2)(r_1 - q_1 E_1) - \frac{r_1}{K} ((r_3 + q_3 E_3)(r_2 + \gamma_2 + q_2 E_2) - \gamma_2 \beta_2) > 0. \tag{5}$$

We assume that there exist a pair of efforts (E_1, E_2, E_3) satisfying (5), (6), and $0 \leq E_i \leq E_{i\max}$. The profit function associates with the equilibrium point $T_3 = (N_1^*, N_2^*, N_3^*)$ is given by

$$\pi(E_1, E_2, E_3) = (p_1 q_1 N_1^*) E_1 + (p_2 q_2 N_2^*) E_2 + (p_3 q_3 N_3^*) E_3 - (c_1 E_1 + c_2 E_2 + c_3 E_3).$$

We need to determine the values of $E_1, E_2,$ and E_3 satisfying the conditions (4), (5), and $0 \leq E_i \leq E_{i\max}$ that maximize profit $\pi(E_1, E_2, E_3)$ and also the equilibrium point $T_3 = (N_1^*, N_2^*, N_3^*)$ remains asymptotically stable.

4 Optimal Harvesting Policy

The biological equilibrium is given by solving the simultaneous equations $\frac{dN_i}{dt} = 0$. The economic equilibrium is said to achieved when the total revenue equals the total cost. The profit function is

$$\pi(E_1, E_2, E_3) = p_1 q_1 N_1 E_1 + p_2 q_2 N_2 E_2 + p_3 q_3 N_3 E_3 - c_1 E_1 - c_2 E_2 - c_3 E_3.$$

The objective is to maximize the present value J of a continuous time-stream of revenues given by

$$J = \int_0^\infty e^{-\delta t} \{ (p_1 q_1 N_1 - c_1) E_1(t) + (p_2 q_2 N_2 - c_2) E_2(t) + (p_3 q_3 N_3 - c_3) E_3(t) \} dt. \tag{6}$$

The symbol δ denotes the instantaneous annual rate of discount. We need to maximize J subject to the constraint equation (3) by using Pontryagin's maximum

principle [15]. The control variables $E_1(t)$, $E_2(t)$, and $E_3(t)$ are subject to the constraints $0 \leq E_i(t) \leq (E_i)_{\max}$ for $i=1,2,3$.

The Hamiltonian equation for this problem is given by

$$H = e^{-\delta t} \{ (p_1 q_1 N_1 - c_1) E_1 + (p_2 q_2 N_2 - c_2) E_2 + (p_3 q_3 N_3 - c_3) E_3 \} \\ + \lambda_1 \{ N_1 (r_1 - q_1 E_1 - r_4 N_1 - \alpha_1 N_3) \} + \lambda_2 \{ (-r_2 - \gamma_2 - q_2 E_2) N_2 + \beta_2 N_3 \} \\ + \lambda_3 \{ (-r_3 - q_3 E_3) N_3 + \gamma_2 N_2 + \alpha_3 N_1 N_3 \}, \quad (7)$$

where $\lambda_1(t)$, $\lambda_2(t)$, and $\lambda_3(t)$ denote the adjoint variables.

The necessary conditions for the control variables E_1 , E_2 , and E_3 to be optimum are $\frac{\partial H}{\partial E_1} = 0$, $\frac{\partial H}{\partial E_2} = 0$, and $\frac{\partial H}{\partial E_3} = 0$. From the Hamiltonian equation

(7), we have

$$\frac{\partial H}{\partial E_1} = e^{-\delta t} (p_1 q_1 N_1 - c_1) - \lambda_1 q_1 N_1 = 0, \\ \frac{\partial H}{\partial E_2} = e^{-\delta t} (p_2 q_2 N_2 - c_2) - \lambda_2 q_2 N_2 = 0, \text{ and} \\ \frac{\partial H}{\partial E_3} = e^{-\delta t} (p_3 q_3 N_3 - c_3) - \lambda_3 q_3 N_3 = 0.$$

From these three equations we get $\lambda_1 = \frac{e^{-\delta t} (p_1 q_1 N_1 - c_1)}{q_1 N_1}$, $\lambda_2 = \frac{e^{-\delta t} (p_2 q_2 N_2 - c_2)}{q_2 N_2}$,

and $\lambda_3 = \frac{e^{-\delta t} (p_3 q_3 N_3 - c_3)}{q_3 N_3}$. From the Hamiltonian equation we also have

$$\frac{\partial H}{\partial N_1} = e^{-\delta t} p_1 q_1 E_1 + \lambda_1 (r_1 - q_1 E_1 - r_4 N_1 - \alpha_1 N_3) - \lambda_1 r_4 N_1 + \lambda_3 \alpha_3 N_3, \\ \frac{\partial H}{\partial N_2} = e^{-\delta t} p_2 q_2 E_2 - \lambda_2 (r_2 + \gamma_2 + q_2 E_2) + \lambda_3 \gamma_2, \text{ and} \\ \frac{\partial H}{\partial N_3} = e^{-\delta t} p_3 q_3 E_3 - \lambda_1 \alpha_1 N_1 + \lambda_2 \beta_2 + \lambda_3 (-r_3 - q_3 E_3 + \alpha_3 N_1).$$

From the Pontryagin's maximum principle $\dot{\lambda}_1 = -\frac{\partial H}{\partial N_1}$, $\dot{\lambda}_2 = -\frac{\partial H}{\partial N_2}$, and

$\dot{\lambda}_3 = -\frac{\partial H}{\partial N_3}$ we get

$$\frac{-\delta e^{-\delta}(p_1 q_1 N_1 - c_1)}{q_1 N_1} + e^{-\delta} p_1 q_1 E_1 + \lambda_1 (r_1 - q_1 E_1 - r_4 N_1 - \alpha_1 N_3) - \lambda_1 r_4 N_1 + \lambda_3 \alpha_3 N_3 = 0, \tag{8}$$

$$\frac{-\delta e^{-\delta}(p_2 q_2 N_2 - c_2)}{q_2 N_2} + e^{-\delta} p_2 q_2 E_2 + \lambda_2 (-r_2 - \gamma_2 - q_2 E_2) + \lambda_3 \gamma_2 = 0, \tag{9}$$

$$\frac{-\delta e^{-\delta}(p_3 q_3 N_3 - c_3)}{q_3 N_3} + e^{-\delta} p_3 q_3 E_3 - \lambda_1 \alpha_1 N_1 + \lambda_2 \beta_2 + \lambda_3 (-r_3 - q_3 E_3 + \alpha_3 N_1) = 0. \tag{10}$$

By substituting $\lambda_1 = \frac{e^{-\delta}(p_1 q_1 N_1 - c_1)}{q_1 N_1}$, $\lambda_2 = \frac{e^{-\delta}(p_2 q_2 N_2 - c_2)}{q_2 N_2}$, and $\lambda_3 = \frac{e^{-\delta}(p_3 q_3 N_3 - c_3)}{q_3 N_3}$ into equations (8), (9), and (10) and then solving them simultaneously, we get E_1 , E_2 , and E_3 , where

$$E_1 = \frac{\delta}{q_1 q_3 (\delta p_1 q_1 N_1 + p_1 q_1 N_1 - \delta c_1)} \{ p_1 q_1 q_3 N_1 - q_3 c_1 - p_1 q_1 q_3 \alpha_1 N_1 N_3 + p_1 q_1 q_3 r_1 N_1 - q_1 \alpha_3 c_3 N_1 - 2 p_1 q_1 q_3 r_4 N_1^2 + q_3 \alpha_1 c_1 N_3 - q_3 r_1 c_1 + 2 q_3 c_1 r_4 N_1 + p_3 q_1 q_3 \alpha_3 N_1 N_3 \}, \tag{11}$$

$$E_2 = \frac{\delta}{q_2 q_3 N_3 (-\delta c_2 + \delta p_2 q_2 N_2 + p_2 q_2 N_2)} \{ p_2 q_2 q_3 N_2 N_3 - q_3 c_2 N_3 + q_3 c_2 r_2 N_3 - p_2 q_2 q_3 r_2 N_2 N_3 - p_2 q_2 q_3 \gamma_2 N_2 N_3 + \gamma_2 p_3 q_2 q_3 N_2 N_3 - \gamma_2 q_2 c_3 N_2 + q_3 c_2 \gamma_2 N_3 \}, \tag{12}$$

$$E_3 = \frac{\delta}{q_1 q_2 q_3 N_2 (-\delta c_3 + \delta p_3 q_3 N_3 + p_3 q_3 N_3)} \{ p_3 q_1 q_2 q_3 N_2 N_3 - q_1 q_2 c_3 N_2 + \alpha_1 c_1 q_2 q_3 N_2 N_3 - \beta_2 q_1 q_3 c_2 N_3 - p_1 q_1 q_2 q_3 \alpha_1 N_1 N_2 N_3 + \beta_2 p_2 q_1 q_2 q_3 N_2 N_3 + q_1 q_2 c_3 r_3 N_2 - p_3 q_1 q_2 q_3 r_3 N_2 N_3 - q_1 q_2 c_3 \alpha_3 N_1 N_2 + p_3 q_1 q_2 q_3 \alpha_3 N_1 N_2 N_3 \}. \tag{13}$$

By substituting $N_1 = N_1^* = \frac{P_5}{\alpha_3 q_5}$, $N_2 = N_2^* = \frac{\beta_2 P_6}{\alpha_3 q_5^2 \alpha_1}$, and

$N_3 = N_3^* = \frac{\beta_2 P_6}{\alpha_3 q_5^2 \alpha_1}$ into (11), (12), and (13), and then solving simultaneously

we get the value of control variables E_1 , E_2 , and E_3 . Therefore, the value of E_1 , E_2 , E_3 , N_1^* , N_2^* , and N_3^* maximize the present value (6).

5 Numerical Examples

In this section we give numerical examples for maximum profit problem and optimal harvesting policy to maximize the present value of revenue.

We consider the set values of the parameters $r_1=1.5$, $r_2=0.2$, $r_3=0.2$, $K=1000$, $\beta_2=0.5$, $\alpha_1=0.001$, $m=0.5$, $\gamma_2=0.05$, $q_1=0.1$, $q_2=0.2$, and $q_3=0.2$ in appropriate units. Take $p_1=1.1$, $p_2=1.2$, $p_3=1.3$, $c_1=0.1$, $c_2=0.2$, and $c_3=0.3$ in appropriate units. Then we have equilibrium point $T_3 = (N_1^*, N_2^*, N_3^*)$, where

$$N_1^* = \frac{200(5 + 8E_2 + 10E_3 + 8E_2E_3)}{5 + 4E_2},$$

$$N_2^* = \frac{1000(60 + 36E_2 - 30E_3 - 24E_2E_3 - 5E_1 - 4E_2E_1)}{25 + 40E_2 + 16E_2^2}, \text{ and}$$

$$N_3^* = \frac{100(60 + 36E_2 - 30E_3 - 24E_2E_3 - 5E_1 - 4E_2E_1)}{5 + 4E_2}.$$

The equilibrium point T_3 is an interior point if E_1 , E_2 , and E_3 satisfy the conditions $5 + 8E_2 + 10E_3 + 8E_2E_3 > 0$ and $(60 + 36E_2 - 30E_3 - 24E_2E_3 - 5E_1 - 4E_2E_1) > 0$. We let $E_{1\max} = 1$, $E_{2\max} = 1$ and $E_{3\max} = 1$, then the equilibrium point T_3 is an interior point when $(E_1, E_2, E_3) \in D_1$, where

$$D_1 = \{(E_1, E_2, E_3) : 0 \leq E_1 \leq 1, 0 \leq E_2 \leq 1, 0 \leq E_3 \leq 1, 5 + 8E_2 + 10E_3 + 8E_2E_3 > 0, (60 + 36E_2 - 30E_3 - 24E_2E_3 - 5E_1 - 4E_2E_1) > 0\}.$$

The profit function associates with the equilibrium point T_3 is given by

$$\pi(E_1, E_2, E_3) = (p_1 q_1 N_1^*) E_1 + (p_2 q_2 N_2^*) E_2 + (p_3 q_3 N_3^*) E_3 - (c_1 E_1 + c_2 E_2 + c_3 E_3).$$

After substituting the values of N_1^*, N_2^*, N_3^* and simplifying we get

$$\begin{aligned} \pi(E_1, E_2, E_3) = & \frac{0.1000}{(5 + 4E_2)^2} \{ 7200E_1E_2E_3 + 2880E_1E_2^2E_3 + 86320E_2^2 + 37080E_2E_3 \\ & + 1160E_1E_2 - 20208E_2^2E_3 - 2576E_1E_2^2 + 4500E_1E_3 - 39000E_3^2 \\ & - 62400E_2E_3^2 - 24960E_2^2E_3^2 + 77925E_3 + 1.4395 \times 10^5 E_2 + 5475E_1 \\ & - 32E_2^2 \}. \end{aligned}$$

By considering the boundary D_1 , we have a critical value of profit function at the boundary D_1 , namely $(E_1^*, E_2^*, E_3^*) = (1, 1, 0.43280)$ that maximizes the profit function with the value $\pi(E_1^*, E_2^*, E_3^*) = 318.47671$. By applying the value of harvesting efforts $(E_1^*, E_2^*, E_3^*) = (1, 1, 0.43280)$ we get equilibrium point $T_3 = (462.0085, 785.5413, 706.9872)$ and the characteristic equation $f(\rho) = \rho^3 + 1.19857\rho^2 + 0.51367\rho + 0.07349$ which has eigenvalues -0.30413 and $-0.44722 \pm 0.20406i$. This means that when we apply the value of efforts $E_1^* = 1$, $E_2^* = 1$, and $E_3^* = 0.43280$, then the three populations with constant efforts of harvesting will remain sustainable for a long time and also maximize the profit function.

Now, we relate the equilibrium point to the maximum present value. Given the set values of parameters $r_1 = 1.5$, $r_2 = 0.2$, $r_3 = 0.3$, $K = 10000$, $\alpha_1 = 0.001$, $m = 0.15$, $\gamma_2 = 0.05$, $\beta_2 = 0.3$, $q_1 = 0.2$, $q_2 = 0.16$, and $q_3 = 0.17$. Take $p_1 = 5$, $p_2 = 10$, $p_3 = 12$, $c_1 = 1.1$, $c_2 = 1.2$, $c_3 = 1.3$, and $\delta = 0.05$ in appropriate units. From these values of parameters we have the optimal equilibrium point $T_3 = (N_1^*, N_2^*, N_3^*)$, where

$$N_1^* = \frac{66.66667}{25 + 16E_2} (600 + 480E_2 + 425E_3 + 272E_2E_3),$$

$$N_2^* = \frac{300}{625 + 800E_2 + 256E_2^2} (3150 + 1920E_2 - 425E_3 - 272E_2E_3 - 500E_1 - 320E_1E_2),$$

$$N_3^* = \frac{10}{25 + 16E_2} (3150 + 1920E_2 - 425E_3 - 272E_2E_3 - 500E_1 - 320E_1E_2).$$

The adjoint variables are

$$\lambda_1 = \frac{-0.000125e^{-0.05t} (1.1992x10^6 + 9.5947x10^5 E_2 + 8.5x10^5 E_3 + 5.44x10^5 E_2E_3)}{600 + 480E_2 + 425E_3 + 272E_2E_3},$$

$$\lambda_2 = \frac{-0.00125e^{-0.05t}}{-3150 - 1920E_2 + 425E_3 + 272E_2E_3 + 500E_1 + 320E_1E_2} \left\{ -1.2594x10^6 - 7.6720x10^5 E_2 + 1.7x10^5 E_3 + 1.088x10^5 E_2E_3 + 2x10^5 E_1 + 1.28x10^5 E_1E_2 + 256E_2^2 \right\},$$

$$\lambda_3 = \frac{-0.00294e^{-0.05t}}{-3150 - 1920E_2 + 425E_3 + 272E_2E_3 + 500E_1 + 320E_1E_2} \left\{ -6.4228x10^5 - 3.9147x10^5 E_2 + 86700E_3 + 55488E_2E_3 + 1.02x10^5 E_1 + 65280E_1E_2 \right\}.$$

Further, we get the optimal harvesting efforts $E_1 = 0.28773096$, $E_2 = 0.24093140$, and $E_3 = 0.13141604$ associate with the optimal equilibrium

point T_3 . From the optimal harvesting efforts we get $T_3 = (N_1^*, N_2^*, N_3^*) = (1802.3766, 1218.6116, 1172.0973)$ with the eigenvalues $-0.16072507 \pm 0.53858755i$ and -0.28943961 . This means that the equilibrium point is locally asymptotically stable. The value of adjoint variables are $\lambda_1 = -0.249847 e^{-0.05t}$, $\lambda_2 = -0.499692 e^{-0.05t}$, and $\lambda_3 = -0.599674 e^{-0.05t}$. Then, we get the maximum value of $J = \int_0^{\infty} 1301.812059 e^{-0.05t} dt = 26036.24118$.

6 Conclusions

The prey-predator population model with constant efforts of harvesting may has an interior equilibrium point $T_3 = (N_1^*, N_2^*, N_3^*)$ which exists when the value of paramaters and harvesting efforts satisfy the conditions $q_5 q_6 - \beta_2 \lambda_2 > 0$, $q_4 \alpha_3 q_5 - p_5 r_4 > 0$ and $0 \leq E_i \leq E_{i\max}$. The interior equilibrium point is possibly asymptotically stable. The existence and stability of the interior equilibrium point $T_3 = (N_1^*, N_2^*, N_3^*)$ depend on the value of parameters and the value of harvesting efforts.

With the restriction $0 \leq E_i \leq E_{i\max}$, there exists a certain condition such that the interior equilibrium point remains stable and also gives maximum profit. There exists a certain value of harvesting efforts and a stable equilibrium point that maximize the present value of revenues by using Pontryagin's maximum principle.

The prey and predator populations can live in coexistence, although the prey, immature predator, and mature predator are harvested with constant effort. Besides that, the populations also give maximum profit and maximum present value of net revenue.

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